**Proof 5**: **SrCWMTExp(A) ≥/=/≤ SrExp(A)**

We want to know for what values of p and q: (1) SrCWMTExp(A) ≥ SrExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

SrExp(A) = -3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5

And

SrCWMTExp(A) = -3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - 3\*p\*q\*\*2 + 6\*p\*q - p + q\*\*2 - 3\*q + 5

Then we can rearrange (1) to produce SrCWMTExp(A) – SrExp(A) ≥ 0 which becomes:

(-3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - 3\*p\*q\*\*2 + 6\*p\*q - p + q\*\*2 - 3\*q + 5) – (-3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5) ≥ 0

When we simplify this, we arrive at the following inequality:

p\*\*2 + 2\*p\*q - p + q\*\*2 – q ≥ 0

This can be factored and simplified again to give the following:

(p+q-1)\*(p+q) ≥ 0

From this we get the solution p+q ≥ 1, meaning that the inequality SrCWMTExp(A) ≥ SrExp(A) is true for all values of p and q where

p+q ≥ 1.

Now let’s see for what values of p and q: (2) SrCWMTExp(A) ≤ SrExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

We now try to see for what values the following is true:

(p+q-1)\*(p+q) ≤ 0

Meaning p+q-1 ≤ 0, which solves to p+q ≤ 1. Thus, the inequality SrCWMTExp(A) ≤ SrExp(A) is true for all values of p and q where

p+q ≤ 1.

Finally, we have that SrCWMTExp(A) = SrExp(A) when (p+q-1)\*(p+q) = 0. This happens only when p+q=0 (i.e.: at the point p=0 , q=0) or p+q=1.

**Proof 6**: **SrCWMTExp(A) ≥/=/≤ CrExp(A)**

We want to know for what values of p and q: (1) SrCWMTExp(A) ≥ CrExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

CrExp(A) = -3\*p\*\*3\*q + 2\*p\*\*3 + 3\*p\*\*2\*q\*\*2 + 2\*p\*\*2\*q - 5\*p\*\*2 - 3\*p\*q\*\*2 + 3\*p\*q + 3\*p + 3

And

SrCWMTExp(A) = -3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - 3\*p\*q\*\*2 + 6\*p\*q - p + q\*\*2 - 3\*q + 5

Then we can rearrange (1) to produce SrCWMTExp(A) – CrExp(A) ≥ 0 which becomes:

(-3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5) - (-3\*p\*\*3\*q + 2\*p\*\*3 + 3\*p\*\*2\*q\*\*2 + 2\*p\*\*2\*q - 5\*p\*\*2 - 3\*p\*q\*\*2 + 3\*p\*q + 3\*p + 3) ≥ 0

When we simplify this, we arrive at the following inequality:

-p\*\*3 - p\*\*2\*q + 3\*p\*\*2 + 3\*p\*q - 4\*p + q\*\*2 - 3\*q + 2 ≥ 0

This can be factored and simplified again to give the following:

(p\*\*2-2\*p-q+2)\*(p+q-1) ≤ 0

From this we get the solution p+q ≤ 1 as well as the solution p=1 and q=1, meaning that the inequality SrCWMTExp(A) ≥ CrExp(A) is true for all values of p and q where p+q ≤ 1 or p=1 and q=1.

Now let’s see for what values of p and q: (2) SrCWMTExp(A) ≤ CrExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

We now try to see for what values the following is true:

-(p\*\*2-2\*p-q+2)\*(p+q-1) ≤ 0 or (p\*\*2-2\*p-q+2)\*(p+q-1) ≥ 0

Meaning p+q-1 ≥ 0, which solves to p+q ≥ 1. Thus, the inequality SrCWMTExp(A) ≤ CrExp(A) is true for all values of p and q where

p+q ≥ 1.

Finally, we have that SrCWMTExp(A) = CrExp(A) when

(p\*\*2-2\*p-q+2)\*(p+q-1) = 0. This happens only when p=1 and q=1, or when p+q=1.

**Proof 7**: **SrCWMTExp(A) ≥/=/≤ TrAExp(A)**

We want to know for what values of p and q: (1) SrCWMTExp(A) ≥ TrAExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

TrAExp(A) = p\*\*4 - 2\*p\*\*3\*q - p\*\*3 + 2\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - p\*q\*\*2 + 3\*p\*q + 2\*p + 3

And

SrCWMTExp(A) = -3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - 3\*p\*q\*\*2 + 6\*p\*q - p + q\*\*2 - 3\*q + 5

Then we can rearrange (1) to produce SrCWMTExp(A) – TrAExp(A) ≥ 0 which becomes:

(-3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5) - (p\*\*4 - 2\*p\*\*3\*q - p\*\*3 + 2\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - p\*q\*\*2 + 3\*p\*q + 2\*p + 3) ≥ 0

When we simplify this, we arrive at the following inequality:

-p\*\*4 - p\*\*3\*q + 2\*p\*\*3 + p\*\*2\*q\*\*2 + p\*q\*\*3 - 2\*p\*q\*\*2 + 3\*p\*q - 3\*p + q\*\*2 - 3\*q + 2 ≥ 0

This can be factored and simplified again to give the following:

-(p+q-1)\*(p\*\*3-p\*\*2-p\*q\*\*2+p\*q-p-q+2) ≥ 0 or

(p+q-1)\*(p\*\*3-p\*\*2-p\*q\*\*2+p\*q-p-q+2) ≤ 0

From this we get the solution p+q ≤ 1, meaning that the inequality SrCWMTExp(A) ≥ TrAExp(A) is true for all values of p and q where

p+q ≤ 1.

Now let’s see for what values of p and q: (2) SrCWMTExp(A) ≤ TrAExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

We now try to see for what values the following is true:

(p+q-1)\*(p\*\*3-p\*\*2-p\*q\*\*2+p\*q-p-q+2) ≥ 0

Meaning p+q-1 ≥ 0, which solves to p+q ≥ 1. Thus, the inequality SrCWMTExp(A) ≤ TrAExp(A) is true for all values of p and q where

p+q ≥ 1.

Finally, we have that SrCWMTExp(A) = TrAExp(A) when

(p+q-1)\*(p\*\*3-p\*\*2-p\*q\*\*2+p\*q-p-q+2) = 0. This happens only when p=1 and q=1, or when p+q=1.

**Proof 8**: **SrExp(A) ≥/=/≤ CrExp(A)**

We want to know for what values of p and q: (1) SrExp(A) ≥ CrExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

CrExp(A) = -3\*p\*\*3\*q + 2\*p\*\*3 + 3\*p\*\*2\*q\*\*2 + 2\*p\*\*2\*q - 5\*p\*\*2 - 3\*p\*q\*\*2 + 3\*p\*q + 3\*p + 3

And

SrExp(A) = -3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5

Then we can rearrange (1) to produce SrExp(A) – CrExp(A) ≥ 0 which becomes:

(-3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5) - (-3\*p\*\*3\*q + 2\*p\*\*3 + 3\*p\*\*2\*q\*\*2 + 2\*p\*\*2\*q - 5\*p\*\*2 - 3\*p\*q\*\*2 + 3\*p\*q + 3\*p + 3) ≥ 0

When we simplify this, we arrive at the following inequality:

-p\*\*3 - p\*\*2\*q + 2\*p\*\*2 + p\*q - 3\*p - 2\*q + 2 ≥ 0

This can be factored and simplified again to give the following:

p+q-1 ≤ 0

From this we get the solution p+q ≤ 1, meaning that the inequality SrExp(A) ≥ CrExp(A) is true for all values of p and q where

p+q ≤ 1.

Now let’s see for what values of p and q: (2) SrExp(A) ≤ CrExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

We now try to see for what values the following is true:

p+q-1 ≥ 0

Meaning p+q ≥ 1. Thus, the inequality SrExp(A) ≤ CrExp(A) is true for all values of p and q where p+q ≥ 1.

Finally, we have that SrExp(A) = CrExp(A) only when p+q = 0.

**Proof 8**: **SrExp(A) ≥/=/≤ TrAExp(A)**

We want to know for what values of p and q: (1) SrExp(A) ≥ TrAExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

TrAExp(A) = p\*\*4 - 2\*p\*\*3\*q - p\*\*3 + 2\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - p\*q\*\*2 + 3\*p\*q + 2\*p + 3

And

SrExp(A) = -3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5

Then we can rearrange (1) to produce SrExp(A) – TrAExp(A) ≥ 0 which becomes:

(-3\*p\*\*3\*q + p\*\*3 + 3\*p\*\*2\*q\*\*2 + p\*\*2\*q - 3\*p\*\*2 - 3\*p\*q\*\*2 + 4\*p\*q - 2\*q + 5) – (p\*\*4 - 2\*p\*\*3\*q - p\*\*3 + 2\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - p\*q\*\*2 + 3\*p\*q + 2\*p + 3) ≥ 0

When we simplify this, we arrive at the following inequality:

-p\*\*4 - p\*\*3\*q + 2\*p\*\*3 + p\*\*2\*q\*\*2 - p\*\*2 + p\*q\*\*3 - 2\*p\*q\*\*2 + p\*q - 2\*p - 2\*q + 2 ≥ 0

This can be factored and simplified again to give the following:

(p+q-1)\*(p\*\*3-p\*\*2-p\*q\*\*2+p\*q+2) ≤ 0

From this we get the solution p+q ≤ 1, meaning that the inequality SrExp(A) ≥ TrAExp(A) is true for all values of p and q where

p+q ≤ 1.

Now let’s see for what values of p and q: (2) SrExp(A) ≤ TrAExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

We now try to see for what values the following is true:

(p+q-1)\*(p\*\*3-p\*\*2-p\*q\*\*2+p\*q+2) ≥ 0

Meaning p+q ≥ 1. Thus, the inequality SrExp(A) ≤ TrAExp(A) is true for all values of p and q where p+q ≥ 1.

Finally, we have that SrExp(A) = TrAExp(A) only when p+q = 0.

**Proof 8**: **CrExp(A) ≥/=/≤ TrAExp(A)**

We want to know for what values of p and q: (1) CrExp(A) ≥ TrAExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

Recall also that:

TrAExp(A) = p\*\*4 - 2\*p\*\*3\*q - p\*\*3 + 2\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - p\*q\*\*2 + 3\*p\*q + 2\*p + 3

And

CrExp(A) = -3\*p\*\*3\*q + 2\*p\*\*3 + 3\*p\*\*2\*q\*\*2 + 2\*p\*\*2\*q - 5\*p\*\*2 - 3\*p\*q\*\*2 + 3\*p\*q + 3\*p + 3

Then we can rearrange (1) to produce CrExp(A) – TrAExp(A) ≥ 0 which becomes:

(-3\*p\*\*3\*q + 2\*p\*\*3 + 3\*p\*\*2\*q\*\*2 + 2\*p\*\*2\*q - 5\*p\*\*2 - 3\*p\*q\*\*2 + 3\*p\*q + 3\*p + 3) – (p\*\*4 - 2\*p\*\*3\*q - p\*\*3 + 2\*p\*\*2\*q\*\*2 + p\*\*2\*q - 2\*p\*\*2 - p\*q\*\*3 - p\*q\*\*2 + 3\*p\*q + 2\*p + 3) ≥ 0

When we simplify this, we arrive at the following inequality:

p\*(-p\*\*3 - p\*\*2\*q + 3\*p\*\*2 + p\*q\*\*2 + p\*q - 3\*p + q\*\*3 - 2\*q\*\*2 + 1) ≥ 0

This can be factored and simplified again to give the following:

(p)\*(p+q-1)(p\*\*2-2p-q\*\*2+q+1) ≤ 0

From this we get the solution p+q ≤ 1, meaning that the inequality CrExp(A) ≥ TrAExp(A) is true for all values of p and q where

p+q ≤ 1.

Now let’s see for what values of p and q: (2) CrExp(A) ≤ TrAExp(A).

Recall that 0 ≤ p ≤ 1 , 0 ≤ q ≤ 1.

We now try to see for what values the following is true:

(p)\*(p+q-1)\*(p\*\*2-2p-q\*\*2+q+1) ≥ 0

Meaning p+q ≥ 1. Thus, the inequality CrExp(A) ≤ TrAExp(A) is true for all values of p and q where p+q ≥ 1.

Finally, we have that SrExp(A) = TrAExp(A) only when p+q = 0, p=1 and q=1, or when p=0 and q=0.